Directions:

1. You have 60 minutes to complete the exam.
2. You are allowed to use a calculator during the exam.
3. Please circle the correct answer(s) in the exam booklet.
4. Some questions ask you to select all answers that apply. These are marked *Indicate all such answers*, and credit is given only when every correct answer is selected and no incorrect one is.
5. Some questions ask you to write a number rather than choose an answer. Write your answer in the box provided.
6. You may show your reasoning in the booklet beside each question. The admissions committee may take your work into account when evaluating your performance.

Good Luck!

Multiple-choice questions:

1. (1 point)

Quantity A	Quantity B
$\frac{3}{8}$ of 240	$\frac{5}{12}$ of 204

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Compute each quantity directly.

$$\text{Quantity A} = \frac{3}{8} \times 240 = 3 \times 30 = 90.$$

$$\text{Quantity B} = \frac{5}{12} \times 204 = 5 \times 17 = 85.$$

Since $90 > 85$, Quantity A is greater.

The correct answer is Choice A.

2. (1 point) A stationery shop sells notebooks at 4 for 9 lari and pens at 3 for 7 lari. Both items are sold only in these bundles.

Quantity A	Quantity B
The cost of 12 notebooks	The cost of 12 pens

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Twelve notebooks are $12 \div 4 = 3$ bundles, so

$$\text{Quantity A} = 3 \times 9 = 27 \text{ lari.}$$

Twelve pens are $12 \div 3 = 4$ bundles, so

$$\text{Quantity B} = 4 \times 7 = 28 \text{ lari.}$$

Since $27 < 28$, Quantity B is greater.

Note that comparing the unit prices gives the same conclusion without finishing the arithmetic: a notebook costs $9/4 = 2.25$ lari and a pen costs $7/3 \approx 2.33$ lari, and the same number of each item is bought.

The correct answer is Choice B.

3. (1 point) $|x - 3| = 5$

Quantity A	Quantity B
x	8

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: An absolute-value equation of this form has two solutions, because $x - 3$ may be either 5 or -5:

$$x - 3 = 5 \quad \Longrightarrow \quad x = 8,$$

$$x - 3 = -5 \quad \Longrightarrow \quad x = -2.$$

If $x = 8$, the two quantities are equal. If $x = -2$, Quantity B is greater. The information given does not fix which solution x is, and the two cases give different comparisons.

The trap here is stopping at the first solution. Whenever a question gives an absolute value, a square, or an even power, check whether a negative branch also satisfies the condition.

The correct answer is Choice D.

4. (1 point) $a \neq 0$

Quantity A	Quantity B
$\dfrac{a^2 + 2a}{a}$	$a + 2$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Factor the numerator of Quantity A:

$$\frac{a^2 + 2a}{a} = \frac{a(a+2)}{a}.$$

Because $a \neq 0$, the factor a cancels, leaving

$$\frac{a(a+2)}{a} = a + 2.$$

This equals Quantity B for every permitted value of a, positive or negative. The condition $a \neq 0$ is what makes the cancellation legitimate and is the only value that had to be excluded.

The two quantities are equal.

The correct answer is Choice C.

5. (1 point)

Quantity A	Quantity B
2^{30}	3^{20}

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: The two powers have different bases and different exponents, so they cannot be compared directly. Rewrite both with a common exponent by using the fact that 30 and 20 share the factor 10:

$$2^{30} = \left(2^3\right)^{10} = 8^{10},$$

$$3^{20} = \left(3^2\right)^{10} = 9^{10}.$$

Both are now tenth powers of positive numbers, and $t \mapsto t^{10}$ is increasing for $t > 0$. Since $8 < 9$, it follows that $8^{10} < 9^{10}$, so $2^{30} < 3^{20}$.

Quantity B is greater.

The correct answer is Choice B.

6. (1 point) The perimeter of a rectangle is 24.

Quantity A	Quantity B
The area of the rectangle	36

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Let the sides be ℓ and w. The perimeter condition gives

$$2(\ell + w) = 24 \quad \Longrightarrow \quad \ell + w = 12,$$

so $w = 12 - \ell$ and the area is

$$A = \ell(12 - \ell).$$

A fixed perimeter does not fix the area. The area is largest when the rectangle is a square:

$$\ell = w = 6 \quad \Longrightarrow \quad A = 36,$$

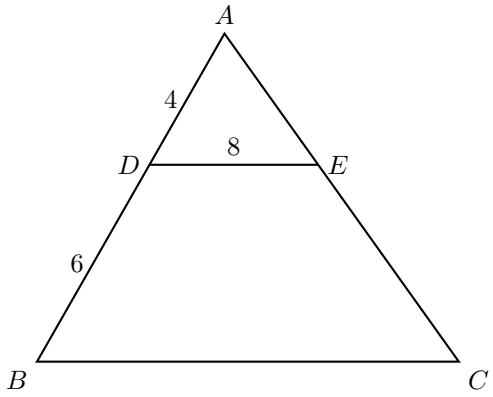
and it shrinks as the rectangle becomes more elongated:

$$\ell = 10,\ w = 2 \quad \Longrightarrow \quad A = 20.$$

So the area can equal 36 or be less than 36. It can never exceed 36, but "the two quantities are equal" is not guaranteed either, since the rectangle is not stated to be a square. Two permitted shapes give different comparisons.

The correct answer is Choice D.

7. (1 point) In triangle ABC below, D lies on AB and E lies on AC, with DE parallel to BC. The lengths are $AD = 4$, $DB = 6$ and $DE = 8$.



Quantity A	Quantity B
The length of BC	18

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Because DE is parallel to BC, triangles ADE and ABC have equal angles and are therefore similar, with AD corresponding to AB and DE to BC.

The scale factor compares the two triangles' sides from the shared vertex A. Note that AB is the *whole* side:

$$AB = AD + DB = 4 + 6 = 10,$$

so

$$\frac{AD}{AB} = \frac{4}{10} = \frac{2}{5}.$$

Applying the same factor to the parallel sides,

$$\frac{DE}{BC} = \frac{2}{5} \quad \Longrightarrow \quad BC = \frac{5}{2} \times 8 = 20.$$

Since $20 > 18$, Quantity A is greater.

The usual error is to use $AD/DB = 4/6$ as the scale factor. That ratio compares the two pieces of one side to each other, not the small triangle to the large one, and it would give $BC = 12$.

The correct answer is Choice A.

8. (1 point) A fair coin is flipped 3 times.

Quantity A	Quantity B
The probability of getting exactly 2 heads	The probability of getting at least 2 heads

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Three flips give $2^3 = 8$ equally likely outcomes. Listing those with at least two heads:

$$HHT, \quad HTH, \quad THH, \quad HHH.$$

Exactly 2 heads happens in 3 of the 8 outcomes, so

$$\text{Quantity A} = \frac{3}{8}.$$

At least 2 heads means exactly 2 or exactly 3, which is 4 of the 8 outcomes, so

$$\text{Quantity B} = \frac{4}{8} = \frac{1}{2}.$$

Since $\frac{3}{8} < \frac{1}{2}$, Quantity B is greater.

The comparison can be settled without any counting: "at least 2 heads" includes every outcome of "exactly 2 heads" and also HHH, so it is strictly more likely.

The correct answer is Choice B.

9. (1 point) The positive integer n has exactly 3 positive divisors.

Quantity A	Quantity B
The number of positive divisors of n^2	5

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: First work out what kind of number n must be.

If n has prime factorisation $p_1^{a_1} p_2^{a_2} \cdots$, its number of divisors is $(a_1 + 1)(a_2 + 1) \cdots$. For that product to equal 3 – a prime – there can be only one factor, and it must be 3. So $a_1 + 1 = 3$, giving

$$n = p^2 \quad \text{for some prime } p.$$

Squaring,

$$n^2 = p^4,$$

whose number of divisors is

$$4 + 1 = 5.$$

The two quantities are equal.

A concrete case makes it plain: $n = 9 = 3^2$ has divisors $1, 3, 9$ – exactly 3 – and $n^2 = 81$ has divisors $1, 3, 9, 27, 81$ – exactly 5. Taking $n = 25$ instead gives 625, again with 5 divisors.

Note that the answer does not depend on which prime p is, which is what makes the relationship determinate rather than Choice D.

The correct answer is Choice C.

10. (1 point) An amount of 480 lari is divided among three people in the ratio $2 : 3 : 5$. How much more does the person with the largest share receive than the person with the smallest share?

A. 48 lari

B. 72 lari

C. 96 lari

D. 120 lari

E. 144 lari

Solution: The ratio $2:3:5$ splits the total into $2+3+5=10$ equal parts, so one part is worth

$$\frac{480}{10} = 48 \text{ lari.}$$

The three shares are therefore

$$2 \times 48 = 96, \qquad 3 \times 48 = 144, \qquad 5 \times 48 = 240.$$

The difference between the largest and smallest is

$$240 - 96 = 144 \text{ lari.}$$

The same answer comes directly from the ratio: the shares differ by $5-2=3$ parts, and $3 \times 48 = 144$.

The correct answer is Choice E.

11. (1 point) What is the value of $\sqrt{72}+\sqrt{50}-\sqrt{32}$?

A. $3\sqrt{2}$

B. $4\sqrt{2}$

C. $5\sqrt{2}$

D. $6\sqrt{2}$

E. $7\sqrt{2}$

Solution: Square roots can only be combined once they share the same radicand, so pull the largest perfect square out of each:

$$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2},$$
$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2},$$
$$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}.$$

All three are now multiples of $\sqrt{2}$, so they combine like terms:

$$6\sqrt{2} + 5\sqrt{2} - 4\sqrt{2} = (6+5-4)\sqrt{2} = 7\sqrt{2}.$$

The correct answer is Choice E.

12. (1 point) If $3(x-2) = 2(x+5) - 4$, what is the value of x?

A. 4

B. 6

C. 10

D. 12

E. 16

Solution: Expand both sides:

$$3x - 6 = 2x + 10 - 4,$$

$$3x - 6 = 2x + 6.$$

Subtract $2x$ from both sides:

$$x - 6 = 6,$$

$$x = 12.$$

Check: $3(12 - 2) = 30$ and $2(12 + 5) - 4 = 34 - 4 = 30$.

The correct answer is Choice D.

13. (1 point) If $9^x = 27^{x-1}$, what is the value of x?

A. 3

B. 4

C. 6

D. 9

E. 12

Solution: Both 9 and 27 are powers of 3, so rewrite each side with base 3:

$$9^x = \left(3^2\right)^x = 3^{2x},$$

$$27^{x-1} = \left(3^3\right)^{x-1} = 3^{3(x-1)} = 3^{3x-3}.$$

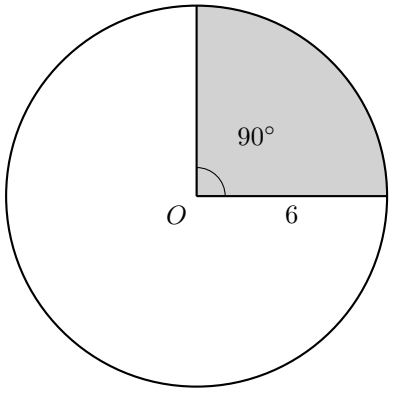
With equal bases, the exponents must be equal:

$$2x = 3x - 3 \quad \Longrightarrow \quad x = 3.$$

Check: $9^3 = 729$ and $27^2 = 729$.

The correct answer is Choice A.

14. (1 point) In the circle below, O is the centre, the radius is 6, and the central angle of the shaded sector is 90°. What is the area of the shaded sector?



A. 3π

B. 4π

C. 6π

D. 8π

E. 9π

Solution: A sector's area is the same fraction of the circle's area as its central angle is of a full turn.

The whole circle has area

$$\pi r^2 = \pi(6)^2 = 36\pi.$$

The sector's central angle is 90° out of 360°, which is one quarter:

$$\frac{90}{360} \times 36\pi = \frac{1}{4} \times 36\pi = 9\pi.$$

Do not confuse this with the arc length, which scales with the circumference rather than the area: a quarter of $2\pi(6) = 12\pi$ is 3π, and that value appears among the choices.

The correct answer is Choice E.

15. (1 point) In the xy-plane, a line passes through the points $(2, 3)$ and $(6, 11)$. At what value of y does the line cross the y-axis?

A. -5

B. -3

C. -1

D. 1

E. 3

Solution: First find the slope from the two given points:

$$m = \frac{11 - 3}{6 - 2} = \frac{8}{4} = 2.$$

Now write the line in the form $y = mx + b$ and use either point to find b. Taking $(2, 3)$:

$$3 = 2(2) + b \quad \Longrightarrow \quad b = 3 - 4 = -1.$$

The line is $y = 2x - 1$, so it crosses the y-axis at $y = -1$.

Check with the other point: $2(6) - 1 = 11$, as required.

Reasoning without algebra gives the same result. Going from $x = 2$ to $x = 0$ means moving 2 units left, and each unit left lowers y by the slope 2, so y falls from 3 to $3 - 4 = -1$.

The correct answer is Choice C.

16. (1 point) At a firm, 40 percent of the employees are engineers and the rest are not. The average salary of the engineers is 60,000 lari, and the average salary across all employees is 48,000 lari. What is the average salary of the employees who are not engineers?

A. 36,000 lari

B. 38,000 lari

C. 40,000 lari

D. 42,000 lari

E. 44,000 lari

Solution: The overall average is a weighted average of the two groups, with weights equal to the groups' shares of the workforce. The head count is never given, and it is not needed – only the proportions matter.

Let x be the average salary of the non-engineers, who make up 60 percent of employees. Then

$$0.40(60{,}000) + 0.60x = 48{,}000.$$

$$24{,}000 + 0.60x = 48{,}000,$$

$$0.60x = 24{,}000 \quad \Longrightarrow \quad x = 40{,}000.$$

A check on the reasoning: the overall average of 48,000 sits 12,000 below the engineers' average and 8,000 above the non-engineers'. Those gaps are in the ratio $12 : 8 = 3 : 2$, the inverse of the group sizes $40 : 60 = 2 : 3$ – the larger group always pulls the average closer to itself.

Averaging the two group figures directly would give $(60{,}000 + 36{,}000)/2$, and reasoning that the overall average must be the midpoint gives 36,000, which appears among the choices. That is right only when the two groups are the same size.

The correct answer is Choice C.

17. (1 point) Of 100 students, 60 play football, 45 play basketball, and 30 play both. Which of the following must be true? Indicate all such statements.

A. Exactly 25 students play neither sport.

B. Exactly 30 students play football but not basketball.

C. More students play basketball only than play neither sport.

D. More than half of the students play at least one of the two sports.

E. The number who play basketball only is twice the number who play neither sport.

Solution: Break the 100 students into the four regions of a Venn diagram.

- Both sports: 30.
- Football only: $60 - 30 = 30$.
- Basketball only: $45 - 30 = 15$.
- At least one sport: $30 + 30 + 15 = 75$.
- Neither: $100 - 75 = 25$.

Choice A. Neither comes to 25. **True.**

Choice B. Football only comes to 30. **True.**

Choice C. Basketball only is 15 and neither is 25, so there are *fewer*, not more. **False.**

Choice D. At least one sport is 75, well above 50. **True.**

Choice E. Twice the number playing neither would be 50, but basketball only is 15. **False.**

The step that does the work is subtracting the overlap. The 60 who play football already include the 30 who play both, so "football only" is 30, not 60.

The correct answer consists of Choices A, B and D.

18. (1 point) In a certain sequence, each term after the first is found by multiplying the preceding term by 3 and then subtracting 4 from the product. The third term of the sequence is 47. What is the first term?

Solution: Write t_1, t_2, t_3 for the first three terms. The rule is

$$t_{n+1} = 3t_n - 4.$$

Work backwards from $t_3 = 47$. From $t_3 = 3t_2 - 4$,

$$47 = 3t_2 - 4 \quad \Longrightarrow \quad 3t_2 = 51 \quad \Longrightarrow \quad t_2 = 17.$$

From $t_2 = 3t_1 - 4$,

$$17 = 3t_1 - 4 \quad \Longrightarrow \quad 3t_1 = 21 \quad \Longrightarrow \quad t_1 = 7.$$

Check by running the rule forwards: $3(7) - 4 = 17$ and $3(17) - 4 = 47$, as required.

The correct answer is 7.

19. (1 point) After a discount of 15 percent, a jacket sells for 289 lari. What was the price of the jacket, in lari, before the discount?

☐ lari

Solution: Let P be the price before the discount. Taking 15 percent off leaves 85 percent of it, so

$$0.85P = 289.$$

Therefore

$$P = \frac{289}{0.85} = 340 \text{ lari.}$$

Check: 15 percent of 340 is $0.15 \times 340 = 51$, and $340 - 51 = 289$.

Note that the discount applies to the original price, so it is wrong to add 15 percent to 289; that would give $289 \times 1.15 = 332.35$, not 340.

The correct answer is 340.

Questions 20–23 refer to the following data.

Preferred Means of Travel to Work, by Age Group

400 respondents, each choosing exactly one means of travel

Age group	Bus	Car	Bicycle	Total
Under 30	70	40	50	160
30 to 49	50	90	20	160
50 and over	30	45	5	80
Total	**150**	**175**	**75**	**400**

20. (1 point) What fraction of the respondents under 30 prefer the bicycle?

A. $\frac{1}{8}$

B. $\frac{2}{9}$

C. $\frac{5}{16}$

D. $\frac{1}{3}$

E. $\frac{2}{3}$

Solution: Restrict to the first row. Of the 160 respondents under 30, 50 prefer the bicycle, so the fraction is

$$\frac{50}{160} = \frac{5}{16}.$$

The denominator is the row total, not the grand total and not the bicycle column total. Using 400 would give $50/400 = 1/8$, and using the 75 bicycle users would give $50/75 = 2/3$; both appear among the choices.

The correct answer is Choice C.

21. (1 point) Of the respondents who prefer the car, approximately what percent are aged 30 to 49?

A. 22.5%

B. 36%

C. 45%

D. 51%

E. 56%

Solution: The group in question is the car column, which holds 175 respondents. Of those, 90 are aged 30 to 49:

$$\frac{90}{175} \approx 0.514 \approx 51\%.$$

Note how the wording fixes the denominator. "Of the respondents who prefer the car" makes the car column the whole group. Had the question asked what percent of the 30-to-49 group prefer the car, the answer would be $90/160 = 56.25\%$, which also appears among the choices.

The correct answer is Choice D.

22. (1 point) How many more respondents aged 30 to 49 prefer the car than prefer the bus?

[ ] respondents

Solution: Both figures come from the middle row of the table, which covers respondents aged 30 to 49. In that row, 90 prefer the car and 50 prefer the bus:

$$90 - 50 = 40.$$

Stay within the row. The column totals, 175 for the car and 150 for the bus, cover all age groups and would give a difference of 25 – the answer to a different question.

The correct answer is 40.

23. (1 point) Which of the following statements are true of the respondents surveyed? Indicate all such statements.

A. More than half of all respondents prefer the car.

B. Among respondents aged 50 and over, the car is the most preferred means of travel.

C. The proportion preferring the bus is greater among respondents under 30 than among those aged 30 to 49.

D. In every age group, fewer than 20 percent of respondents prefer the bicycle.

E. The number of respondents under 30 equals the number aged 30 to 49.

Solution: Choice A. The car column totals 175 out of 400 respondents:

$$\frac{175}{400} = 43.75\%,$$

which is below half. The car is the most popular single option without being the choice of a majority. **False.**

Choice B. In the bottom row, 45 prefer the car against 30 for the bus and 5 for the bicycle. **True.**

Choice C. These are proportions within each row, and both rows happen to total 160:

$$\text{under } 30: \ \frac{70}{160} = 43.75\%, \qquad 30 \text{ to } 49: \ \frac{50}{160} = 31.25\%.$$

True.

Choice D. Among those under 30 the bicycle share is

$$\frac{50}{160} = 31.25\%,$$

well above 20 percent. **False.**

Choice E. Both row totals are 160. **True.**

Choices A and D are each defeated by comparing against the right base. In a two-way table, always settle first whether a claim is about a row, a column, or the whole survey – the same three numbers support very different percentages depending on the denominator chosen.

The correct answer consists of Choices B, C and E.

The exam booklet ends here.

Question	Points	Score
1	1	
2	1	
3	1	
4	1	
5	1	
6	1	
7	1	
8	1	
9	1	
10	1	
11	1	
12	1	
13	1	
14	1	
15	1	
16	1	
17	1	
18	1	
19	1	
20	1	
21	1	
22	1	
23	1	
Total:	23	

1. Ans: (A)
2. Ans: (B)
3. Ans: (D)
4. Ans: (C)
5. Ans: (B)
6. Ans: (D)
7. Ans: (A)
8. Ans: (B)
9. Ans: (C)
10. Ans: (E)
11. Ans: (E)
12. Ans: (D)
13. Ans: (A)
14. Ans: (E)
15. Ans: (C)
16. Ans: (C)
17. Ans: (A), (B), (D)
18. Ans: (7)
19. Ans: (340)
20. Ans: (C)
21. Ans: (D)
22. Ans: (40)
23. Ans: (B), (C), (E)