Entry Exam – ISET

Directions:

1. You have 90 minutes to complete the exam.
2. You are allowed to have a calculator during the exam.
3. Please, circle correct answer(s) on the exam booklet.
4. Some exam questions might ask you to choose all answers that apply.
5. You can show your reasoning in the exam booklet next to the spaces in the question. admissions committee might consider your attempts while evaluating your performance on the exam.

Good Luck!

Multiple-choice questions:

1. (1 point) Points R, S, and T lie on a number line, where S is between R and T. The distance between R and S is 6, and the distance between R and T is 15.

Quantity A	**Quantity B**
The distance between the midpoints of line segments RS and ST	The distance between S and T

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Let the points be on a number line. Since S is between R and T, the distance between S and T is the difference between the distance RT and the distance RS.

$$\text{Distance } ST = \text{Distance } RT - \text{Distance } RS = 15 - 6 = 9$$

So, Quantity B is 9.

To find Quantity A, we first find the lengths of the segments RS and ST. We are given that the length of RS is 6. We calculated the length of ST to be 9. The midpoint of RS is at a distance of $\frac{1}{2} \times RS = \frac{1}{2} \times 6 = 3$ from R (or S). The midpoint of ST is at a distance of $\frac{1}{2} \times ST = \frac{1}{2} \times 9 = 4.5$ from S (or T).

Let's place R at coordinate 0. Then S is at coordinate 6, and T is at coordinate 15. The midpoint of RS (segment from 0 to 6) is at coordinate $\frac{0+6}{2} = 3$. The midpoint of ST (segment from 6 to 15) is at coordinate $\frac{6+15}{2} = \frac{21}{2} = 10.5$. The distance between these two midpoints is the absolute difference of their coordinates:

$$\text{Distance between midpoints} = |10.5 - 3| = 7.5$$

Alternatively, the distance between the midpoint of RS and the midpoint of ST is half the length of RS plus half the length of ST:

$$\text{Distance} = \frac{1}{2}RS + \frac{1}{2}ST = \frac{6}{2} + \frac{9}{2} = 3 + 4.5 = 7.5$$

So, Quantity A is 7.5.

Comparing the two quantities: Quantity A = 7.5 Quantity B = 9 Since $7.5 < 9$, Quantity B is greater.

The correct answer is Choice B.

2. (1 point) S is a set of 8 numbers, of which 4 are negative and 4 are positive.

Quantity A	**Quantity B**
The average (arithmetic mean) of the numbers in S	The median of the numbers in S

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: The only information given is that the set S contains 8 numbers: 4 negative and 4 positive. The specific values of these numbers are not provided. Sets with 4 negative and 4 positive numbers can vary greatly, which may affect the average and median differently. Therefore, it is likely that the relationship between Quantity A (the average) and Quantity B (the median) cannot be determined.

To confirm this, consider some examples:

Example 1: Let $S = \{-4, -3, -2, -1, 1, 2, 3, 4\}$. The numbers are already ordered. Since there are 8 numbers, the median is the average of the 4th and 5th numbers:

$$\text{Median} = \frac{-1+1}{2} = \frac{0}{2} = 0$$

The sum of the numbers is $(-4-3-2-1)+(1+2+3+4) = -10+10 = 0$. The average is the sum divided by the count (8):

$$\text{Average} = \frac{0}{8} = 0$$

In this case, Quantity A = Quantity B.

Example 2: Let $S = \{-100, -3, -2, -1, 1, 2, 3, 4\}$. The numbers are ordered. The median is still the average of the 4th and 5th numbers:

$$\text{Median} = \frac{-1+1}{2} = 0$$

The sum of the numbers is $(-100-3-2-1)+(1+2+3+4) = -106+10 = -96$. The average is:

$$\text{Average} = \frac{-96}{8} = -12$$

In this case, Average $(-12) <$ Median (0), so Quantity B $>$ Quantity A.

Example 3: Let $S = \{-4, -3, -2, -1, 1, 2, 3, 100\}$. The numbers are ordered. The median is still:

$$\text{Median} = \frac{-1+1}{2} = 0$$

The sum of the numbers is $(-4-3-2-1)+(1+2+3+100) = -10+106 = 96$. The average is:

$$\text{Average} = \frac{96}{8} = 12$$

In this case, Average $(12) >$ Median (0), so Quantity A $>$ Quantity B.

Since we found cases where A = B, B > A, and A > B, the relationship between Quantity A and Quantity B cannot be determined from the information given.

The correct answer is Choice D.

3. (1 point) The length of each side of rectangle R is an integer, and the area of R is 36.

Quantity A	**Quantity B**
The number of possible values of the perimeter of R	6

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: Let the length and width of the rectangle R be l and w, respectively. We are given that l and w are integers and the area is $l \times w = 36$. We need to find all possible pairs of integers (l, w) whose product is 36. Since l and w represent lengths, they must be positive integers. The order of l and w does not matter for the dimensions of the rectangle (e.g., 4×9 is the same rectangle as 9×4). The pairs of factors are:

- 1×36

- 2×18
- 3×12
- 4×9
- 6×6

These are the 5 possible dimensions for rectangle R. Now, we calculate the perimeter $P = 2(l + w)$ for each pair:

- For 1×36: $P = 2(1 + 36) = 2(37) = 74$
- For 2×18: $P = 2(2 + 18) = 2(20) = 40$
- For 3×12: $P = 2(3 + 12) = 2(15) = 30$
- For 4×9: $P = 2(4 + 9) = 2(13) = 26$
- For 6×6: $P = 2(6 + 6) = 2(12) = 24$

The possible values for the perimeter of R are $\{74, 40, 30, 26, 24\}$. Quantity A is the number of possible values of the perimeter. Counting the distinct values in the set, we find there are 5 possible values. So, Quantity A = 5.

Quantity B is 6.

Comparing the two quantities: Quantity A = 5 Quantity B = 6 Since $5 < 6$, Quantity B is greater.

The correct answer is Choice B.

4. (1 point) Given the equations:

$$x = (z - 1)^2$$
$$y = (z + 1)^2$$

Quantity A	**Quantity B**
The average (arithmetic mean) of x and y	z^2

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.

Solution: Quantity A is the average (arithmetic mean) of x and y. The formula for the average is $\frac{x+y}{2}$. We are given $x = (z - 1)^2$ and $y = (z + 1)^2$. Substitute these into the average formula:

$$\text{Quantity A} = \frac{(z - 1)^2 + (z + 1)^2}{2}$$

Now, expand the squared terms in the numerator:

$$(z - 1)^2 = z^2 - 2z + 1$$
$$(z + 1)^2 = z^2 + 2z + 1$$

Substitute these expanded forms back into the expression for Quantity A:

$$\text{Quantity A} = \frac{(z^2 - 2z + 1) + (z^2 + 2z + 1)}{2}$$

Simplify the numerator by combining like terms:

$$\text{Quantity A} = \frac{z^2 + z^2 - 2z + 2z + 1 + 1}{2} = \frac{2z^2 + 2}{2}$$

Factor out a 2 from the numerator and simplify:

$$\text{Quantity A} = \frac{2(z^2 + 1)}{2} = z^2 + 1$$

So, Quantity A is equal to $z^2 + 1$. Quantity B is z^2.

Now compare Quantity A and Quantity B: Quantity A $= z^2 + 1$ Quantity B $= z^2$

Since z^2 is always non-negative for any real number z, $z^2 + 1$ will always be 1 greater than z^2. Therefore, $z^2 + 1 > z^2$ for all values of z. Quantity A is always greater than Quantity B.

The correct answer is Choice A.

5. (1 point) x, y, and z are the lengths of the sides of a triangle.

Quantity A	**Quantity B**
$x + y + z$	$2z$

- **A. Quantity A is greater.**
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

Solution: We are asked to compare Quantity A ($x+y+z$) with Quantity B ($2z$). A useful strategy for quantitative comparison questions is to simplify the comparison. We can subtract z from both quantities without changing the relationship between them. Comparing $x+y+z$ and $2z$ is equivalent to comparing $(x+y+z)-z$ and $2z-z$. This simplifies to comparing $x+y$ and z.

The premise states that x, y, and z are the lengths of the sides of a triangle. The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the remaining side. Applying this theorem to sides x and y, we have:

$$x + y > z$$

Since $x + y$ is greater than z, it follows that the original Quantity A ($x + y + z$) is greater than the original Quantity B ($2z$).

Therefore, Quantity A is greater than Quantity B.

The correct answer is Choice A.

6. (1 point) At a club meeting, there are 10 more club members than nonmembers. The number of club members at the meeting is c.

Quantity A	**Quantity B**
The total number of people at the club meeting	$2c - 10$

- A. Quantity A is greater.
- B. Quantity B is greater.
- **C. The two quantities are equal.**
- D. The relationship cannot be determined from the information given.

Solution: Let c be the number of club members and let n be the number of nonmembers. We are given that the number of club members is c. We are also given that there are 10 more club members than nonmembers. This can be written as an equation:

$$c = n + 10$$

To find the number of nonmembers (n) in terms of c, we rearrange the equation:

$$n = c - 10$$

Quantity A is the total number of people at the meeting, which is the sum of the club members and the nonmembers:

$$\text{Total people} = \text{members} + \text{nonmembers} = c + n$$

Now substitute the expression for n in terms of c:

$$\text{Quantity A} = c + (c - 10)$$

Simplify the expression:

$$\text{Quantity A} = 2c - 10$$

Quantity B is given as $2c - 10$.

Comparing the two quantities: Quantity A $= 2c - 10$ Quantity B $= 2c - 10$

Since both quantities are equal to $2c - 10$, they are equal.

The correct answer is Choice C.

7. (1 point) n is a positive integer that is greater than 3 and has d positive divisors.

Quantity A	**Quantity B**
n	2^{d-1}

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal.
D. The relationship cannot be determined from the information given.

Solution: We are given that n is an integer greater than 3, and d is the number of positive divisors of n. We need to compare n (Quantity A) with 2^{d-1} (Quantity B). Since the relationship is not immediately obvious, we can test a few values of n that satisfy the condition $n > 3$.

Case 1: Let $n = 4$. The positive divisors of 4 are 1, 2, and 4. The number of divisors is $d = 3$. Quantity A is $n = 4$. Quantity B is $2^{d-1} = 2^{3-1} = 2^2 = 4$. In this case, Quantity A = Quantity B.

Case 2: Let $n = 5$. Since 5 is a prime number, its positive divisors are 1 and 5. The number of divisors is $d = 2$. Quantity A is $n = 5$. Quantity B is $2^{d-1} = 2^{2-1} = 2^1 = 2$. In this case, Quantity A (5) is greater than Quantity B (2). So, Quantity A > Quantity B.

Case 3: Let $n = 6$. The positive divisors of 6 are 1, 2, 3, and 6. The number of divisors is $d = 4$. Quantity A is $n = 6$. Quantity B is $2^{d-1} = 2^{4-1} = 2^3 = 8$. In this case, Quantity A (6) is less than Quantity B (8). So, Quantity A < Quantity B.

Since we found cases where Quantity A = Quantity B, Quantity A > Quantity B, and Quantity A < Quantity B, the relationship between the two quantities cannot be determined from the information given.

The correct answer is Choice D.

8. (1 point) Given $m = 10^{32} + 2$. When m is divided by 11, the remainder is r.

Quantity A	**Quantity B**
r	3

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: We need to find the remainder r when $m = 10^{32} + 2$ is divided by 11. This is equivalent to finding the value of $m \pmod{11}$. We can use properties of modular arithmetic. First, find the remainder of 10 when divided by 11:

$$10 \div 11 = 0 \text{ remainder } 10$$

In modular arithmetic notation, this is $10 \equiv 10 \pmod{11}$. It's often easier to work with negative remainders if they are smaller in absolute value. Since $10 = 11 - 1$, we can also write:

$$10 \equiv -1 \pmod{11}$$

Now we can find the remainder of 10^{32} when divided by 11:

$$10^{32} \equiv (-1)^{32} \pmod{11}$$

Since 32 is an even exponent, $(-1)^{32} = 1$.

$$10^{32} \equiv 1 \pmod{11}$$

Now we can find the remainder of the entire expression $m = 10^{32} + 2$:

$$m = 10^{32} + 2 \equiv 1 + 2 \pmod{11}$$

$$m \equiv 3 \pmod{11}$$

The remainder r when m is divided by 11 is 3. So, Quantity A is $r = 3$.

Quantity B is given as 3.

Comparing the two quantities: Quantity A = 3 Quantity B = 3 The two quantities are equal.

Alternatively, one could look for a pattern in the remainders of $10^n + 2$ when divided by 11:

- $n = 1$: $10^1 + 2 = 12$. $12 \equiv 1 \pmod{11}$.
- $n = 2$: $10^2 + 2 = 102$. $102 = 9 \times 11 + 3$. $102 \equiv 3 \pmod{11}$.
- $n = 3$: $10^3 + 2 = 1002$. $1002 = 91 \times 11 + 1$. $1002 \equiv 1 \pmod{11}$.
- $n = 4$: $10^4 + 2 = 10002$. $10002 = 909 \times 11 + 3$. $10002 \equiv 3 \pmod{11}$.

The pattern of remainders is 1, 3, 1, 3, ... The remainder is 1 when n is odd and 3 when n is even. Since $n = 32$ is even, the remainder r is 3. Quantity A = 3, Quantity B = 3. They are equal.

The correct answer is Choice C.

9. (1 point) Given the equations:

$$xy = 8$$

$$x = y - 2$$

Quantity A	Quantity B
y	0

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

Solution: We are given a system of two equations with two variables, x and y: 1) $xy = 8$ 2) $x = y - 2$ We need to determine the value(s) of y to compare Quantity A (y) with Quantity B (0). We can substitute the expression for x from the second equation into the first equation:

$$(y-2)y = 8$$

Expand the left side:

$$y^2 - 2y = 8$$

Rearrange the equation into standard quadratic form ($ay^2 + by + c = 0$):

$$y^2 - 2y - 8 = 0$$

We can solve this quadratic equation for y by factoring. We need two numbers that multiply to -8 and add to -2. These numbers are -4 and 2.

$$(y-4)(y+2) = 0$$

This equation holds true if either factor is zero:

$$y - 4 = 0 \quad \Rightarrow \quad y = 4$$

$$y + 2 = 0 \quad \Rightarrow \quad y = -2$$

So, there are two possible values for y: 4 and -2.

Now compare Quantity A (y) with Quantity B (0): **Case 1:** If $y = 4$. Quantity A = 4. Quantity B = 0. In this case, Quantity A > Quantity B.

Case 2: If $y = -2$. Quantity A = -2. Quantity B = 0. In this case, Quantity A < Quantity B.

Since Quantity A can be greater than Quantity B in one case and less than Quantity B in another case, the relationship between Quantity A and Quantity B cannot be determined from the information given.

The correct answer is Choice D.

10. (1 point) The area of circle W is 16π and the area of circle Z is 4π. What is the ratio of the circumference of W to the circumference of Z?

A. 2 to 1

B. 4 to 1

C. 8 to 1

D. 16 to 1

E. 32 to 1

Solution: Let r_W be the radius of circle W and r_Z be the radius of circle Z. The formula for the area of a circle with radius r is $A = \pi r^2$. The formula for the circumference of a circle with radius r is $C = 2\pi r$.

For circle W: The area is given as $A_W = 16\pi$. Using the area formula: $\pi r_W^2 = 16\pi$. Divide both sides by π: $r_W^2 = 16$. Take the square root (radius must be positive): $r_W = \sqrt{16} = 4$. The circumference of circle W is $C_W = 2\pi r_W = 2\pi(4) = 8\pi$.

For circle Z: The area is given as $A_Z = 4\pi$. Using the area formula: $\pi r_Z^2 = 4\pi$. Divide both sides by π: $r_Z^2 = 4$. Take the square root: $r_Z = \sqrt{4} = 2$. The circumference of circle Z is $C_Z = 2\pi r_Z = 2\pi(2) = 4\pi$.

The question asks for the ratio of the circumference of W to the circumference of Z.

$$\text{Ratio} = \frac{C_W}{C_Z} = \frac{8\pi}{4\pi}$$

Cancel π and simplify the fraction:

$$\text{Ratio} = \frac{8}{4} = \frac{2}{1}$$

The ratio is 2 to 1.

The correct answer is Choice A.

11. (1 point) In the xy-plane, a quadrilateral has vertices at $(-1, 4)$, $(7, 4)$, $(7, -5)$, and $(-1, -5)$. What is the perimeter of the quadrilateral?

A. 17

B. 18

C. 19

D. 32

E. 34

Solution: Let the vertices be $A = (-1, 4)$, $B = (7, 4)$, $C = (7, -5)$, and $D = (-1, -5)$. We can find the lengths of the sides of the quadrilateral by calculating the distance between consecutive vertices.

Length of side AB (connecting $(-1, 4)$ and $(7, 4)$): Since the y-coordinates are the same, this is a horizontal line. The length is the absolute difference of the x-coordinates:

$$\text{Length}_{AB} = |7 - (-1)| = |7 + 1| = 8$$

Length of side BC (connecting $(7, 4)$ and $(7, -5)$): Since the x-coordinates are the same, this is a vertical line. The length is the absolute difference of the y-coordinates:

$$\text{Length}_{BC} = |4 - (-5)| = |4 + 5| = 9$$

Length of side CD (connecting $(7, -5)$ and $(-1, -5)$): Since the y-coordinates are the same, this is a horizontal line. The length is the absolute difference of the x-coordinates:

$$\text{Length}_{CD} = |7 - (-1)| = |7 + 1| = 8$$

Length of side DA (connecting $(-1, -5)$ and $(-1, 4)$): Since the x-coordinates are the same, this is a vertical line. The length is the absolute difference of the y-coordinates:

$$\text{Length}_{DA} = |-5 - 4| = |-9| = 9$$

The quadrilateral has two sides of length 8 and two sides of length 9. This confirms it is a rectangle. The perimeter is the sum of the lengths of all four sides:

$$\text{Perimeter} = \text{Length}_{AB} + \text{Length}_{BC} + \text{Length}_{CD} + \text{Length}_{DA}$$

$$\text{Perimeter} = 8 + 9 + 8 + 9$$

$$\text{Perimeter} = 2 \times 8 + 2 \times 9 = 16 + 18 = 34$$

Alternatively, using the formula for the perimeter of a rectangle $P = 2(l + w)$:

$$P = 2(8 + 9) = 2(17) = 34$$

The correct answer is Choice E.

12. (1 point) **DISTRIBUTION OF THE HEIGHTS OF 80 STUDENTS**

Height (centimeters)	**Number of Students**
140–144	6
145–149	26
150–154	32
155–159	12
160–164	4
Total	**80**

The table above shows the frequency distribution of the heights of 80 students. What is the least possible range of the heights of the 80 students?

A. 15

B. 16

C. 20

D. 24

E. 28

Solution: The range of a set of data is the difference between the greatest value and the least value in the set.

$$\text{Range} = \text{Maximum Value} - \text{Minimum Value}$$

The table provides height intervals for the 80 students. We need to find the *least possible* range.

The minimum height of any student must be within the lowest interval, 140–144 cm. The maximum height of any student must be within the highest interval, 160–164 cm.

To find the least possible range, we need to minimize the difference between the maximum and minimum heights. This occurs when the maximum height is as small as possible and the minimum height is as large as possible.

The smallest possible maximum height occurs if the tallest student(s) are at the lower end of the highest interval. The lowest value in the 160–164 cm interval is 160 cm. So, the minimum possible maximum height is 160 cm.

The largest possible minimum height occurs if the shortest student(s) are at the upper end of the lowest interval. The highest value in the 140–144 cm interval is 144 cm. So, the maximum possible minimum height is 144 cm.

The least possible range is the difference between the minimum possible maximum height and the maximum possible minimum height:

$$\text{Least Possible Range} = (\text{Smallest possible max value}) - (\text{Largest possible min value})$$

$$\text{Least Possible Range} = 160 \text{ cm} - 144 \text{ cm} = 16 \text{ cm}$$

The correct answer is Choice B.

13. (1 point) Which of the following functions f defined for all numbers x has the property that $f(-x) = -f(x)$ for all numbers x?

A. $f(x) = \frac{x^3}{x^2+1}$

B. $f(x) = \frac{x^2-1}{x^2+1}$

C. $f(x) = x^2(x^2 - 1)$

D. $f(x) = x(x^3 - 1)$

E. $f(x) = x^2(x^3 - 1)$

Solution: The property $f(-x) = -f(x)$ defines an odd function. We need to check which of the given functions satisfies this property. Let's test each choice.

Choice A: $f(x) = \frac{x^3}{x^2+1}$ First, find $f(-x)$ by replacing x with $-x$:

$$f(-x) = \frac{(-x)^3}{(-x)^2+1} = \frac{-x^3}{x^2+1}$$

Next, find $-f(x)$ by multiplying the original function by -1:

$$-f(x) = -\left(\frac{x^3}{x^2+1}\right) = \frac{-x^3}{x^2+1}$$

Since $f(-x) = \frac{-x^3}{x^2+1}$ and $-f(x) = \frac{-x^3}{x^2+1}$, we have $f(-x) = -f(x)$. Therefore, Choice A satisfies the property. Since the question implies there is only one correct answer among the choices, we have found it.

(Optional verification of other choices): **Choice B:** $f(x) = \frac{x^2-1}{x^2+1}$. $f(-x) = \frac{(-x)^2-1}{(-x)^2+1} = \frac{x^2-1}{x^2+1} = f(x)$. This is an even function ($f(-x) = f(x)$). **Choice C:** $f(x) = x^2(x^2-1) = x^4-x^2$. $f(-x) = (-x)^4-(-x)^2 = x^4-x^2 = f(x)$. This is an even function. **Choice D:** $f(x) = x(x^3-1) = x^4-x$. $f(-x) = (-x)^4-(-x) = x^4+x$. This is neither even nor odd. **Choice E:** $f(x) = x^2(x^3-1) = x^5-x^2$. $f(-x) = (-x)^5-(-x)^2 = -x^5-x^2$. This is neither even nor odd.

The correct answer is Choice A.

14. (1 point) If 10^x equals 0.1 percent of 10^y, where x and y are integers, which of the following must be true?

A. $y = x + 2$
B. $y = x + 3$
C. $x = y + 3$
D. $y = 1,000x$
E. $x = 1,000y$

Solution: First, let's express "0.1 percent" as a fraction or a power of 10.

$$0.1 \text{ percent} = \frac{0.1}{100} = \frac{1}{1000} = 10^{-3}$$

The problem states that "10^x equals 0.1 percent of 10^y". We can write this as an equation:

$$10^x = (0.1 \text{ percent}) \times 10^y$$

Substitute the value of 0.1 percent we found:

$$10^x = 10^{-3} \times 10^y$$

Using the rule of exponents $a^m \times a^n = a^{m+n}$ on the right side:

$$10^x = 10^{-3+y}$$

$$10^x = 10^{y-3}$$

Since the bases are both 10, the exponents must be equal:

$$x = y - 3$$

We need to find which answer choice is equivalent to this equation. Let's rearrange the equation to solve for y: Add 3 to both sides:

$$x + 3 = y$$

This is equivalent to $y = x + 3$.

Comparing this result to the answer choices, it matches Choice B. Choice A: $y = x + 2$ (Incorrect) Choice B: $y = x + 3$ (Correct) Choice C: $x = y + 3$ (Equivalent to $y = x - 3$, Incorrect) Choice D: $y = 1000x$ (Incorrect) Choice E: $x = 1000y$ (Incorrect)

The correct answer is Choice B.

15. (1 point) A car dealer received a shipment of cars, half of which were black, with the remainder consisting of equal numbers of blue, silver, and white cars. During the next month, 70 percent of the black cars, 80 percent of the blue cars, 30 percent of the silver cars, and 40 percent of the white cars were sold. What percent of the cars in the shipment were sold during that month?

A. 36%
B. 50%
C. 55%
D. 60%
E. 72%

Solution: First, determine the fraction of the total shipment represented by each color of car.

- Fraction of black cars $= \frac{1}{2}$.
- The remainder of the cars is $1 - \frac{1}{2} = \frac{1}{2}$.
- This remainder consists of equal numbers of blue, silver, and white cars. So, the fraction for each of these three colors is $\frac{1}{3}$ of the remainder:

$$\text{Fraction Blue} = \text{Fraction Silver} = \text{Fraction White} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Check: $\frac{1}{2}(\text{black}) + \frac{1}{6}(\text{blue}) + \frac{1}{6}(\text{silver}) + \frac{1}{6}(\text{white}) = \frac{3}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$.
Next, calculate the percent of the total shipment that was sold. This is the sum of the percentages sold for each color, weighted by the fraction of that color in the shipment.

Percent Sold = (Fraction Black×% Black Sold)+(Fraction Blue×% Blue Sold)+(Fraction Silver×% Silver Sold)+(Fraction W

$$\text{Percent Sold} = \left(\frac{1}{2}\right)(70\%) + \left(\frac{1}{6}\right)(80\%) + \left(\frac{1}{6}\right)(30\%) + \left(\frac{1}{6}\right)(40\%)$$

It's easier to work with decimals or fractions for the percentages: $70\% = 0.7$, $80\% = 0.8$, $30\% = 0.3$, $40\% = 0.4$.

$$\text{Total Fraction Sold} = \left(\frac{1}{2}\right)(0.7) + \left(\frac{1}{6}\right)(0.8) + \left(\frac{1}{6}\right)(0.3) + \left(\frac{1}{6}\right)(0.4)$$

$$\text{Total Fraction Sold} = 0.35 + \frac{0.8 + 0.3 + 0.4}{6}$$

$$\text{Total Fraction Sold} = 0.35 + \frac{1.5}{6}$$

$$\text{Total Fraction Sold} = 0.35 + 0.25$$

$$\text{Total Fraction Sold} = 0.60$$

To express this as a percentage, multiply by 100%:

$$\text{Percent Sold} = 0.60 \times 100\% = 60\%$$

Alternatively, keeping percentages:

$$\text{Percent Sold} = \left(\frac{1}{2}\right)(70\%) + \frac{1}{6}(80\% + 30\% + 40\%)$$

$$\text{Percent Sold} = 35\% + \frac{1}{6}(150\%)$$

$$\text{Percent Sold} = 35\% + 25\%$$

$$\text{Percent Sold} = 60\%$$

The correct answer is Choice D.

16. (1 point) If an investment of P dollars is made today and the value of the investment doubles every 7 years, what will be the value of the investment, in dollars, 28 years from today?

A. $8P^4$

B. P^4

C. $16P$

D. $8P$

E. $4P$

Solution: The initial investment is P dollars. The value of the investment doubles every 7 years. We want to find the value after 28 years.

First, determine how many doubling periods occur in 28 years.

$$\text{Number of doubling periods} = \frac{\text{Total time}}{\text{Doubling time per period}} = \frac{28 \text{ years}}{7 \text{ years/period}} = 4 \text{ periods}$$

The value of the investment is multiplied by 2 for each doubling period. Since there are 4 doubling periods, the initial value P will be multiplied by 2, four times.

The final value can be calculated using the formula:

$$\text{Final Value} = \text{Initial Value} \times (2)^{\text{number of doubling periods}}$$

$$\text{Final Value} = P \times 2^4$$

Calculate 2^4:

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

Substitute this back into the formula for the final value:

$$\text{Final Value} = P \times 16 = 16P$$

Alternatively, we can track the value step-by-step:

- Year 0: Value $= P$
- Year 7: Value $= P \times 2 = 2P$
- Year 14: Value $= 2P \times 2 = 4P$
- Year 21: Value $= 4P \times 2 = 8P$
- Year 28: Value $= 8P \times 2 = 16P$

Both methods show that the value of the investment after 28 years will be $16P$.

The correct answer is Choice C.

17. (1 point) In a certain sequence of numbers, each term after the first term is found by multiplying the preceding term by 2 and then subtracting 3 from the product. If the 4th term in the sequence is 19, which of the following numbers are in the sequence?

Indicate all such numbers.

A. [A] 5

B. [B] 8

C. [C] 11

D. [D] 16

E. [E] 22

F. [F] 35

Solution: Let the sequence be denoted by a_n, where n is the term number. The rule for the sequence is $a_n = 2a_{n-1} - 3$ for $n > 1$. We are given the 4th term, $a_4 = 19$.

We can find the next term(s) using the rule directly:

$$a_5 = 2a_4 - 3 = 2(19) - 3 = 38 - 3 = 35$$

We can find the preceding terms by rearranging the rule to solve for a_{n-1}: $a_n = 2a_{n-1} - 3$ $a_n + 3 = 2a_{n-1}$

$$a_{n-1} = \frac{a_n + 3}{2}$$

Now, let's find the terms before a_4:

$$a_3 = \frac{a_4 + 3}{2} = \frac{19 + 3}{2} = \frac{22}{2} = 11$$

$$a_2 = \frac{a_3 + 3}{2} = \frac{11 + 3}{2} = \frac{14}{2} = 7$$

$$a_1 = \frac{a_2 + 3}{2} = \frac{7 + 3}{2} = \frac{10}{2} = 5$$

So, the first five terms of the sequence are 5, 7, 11, 19, 35.

Now we check which of the given options are in this list:

A 5 is in the sequence (a_1).

B 8 is not in the sequence.

C 11 is in the sequence (a_3).

D 16 is not in the sequence.

E 22 is not in the sequence.

F 35 is in the sequence (a_5).

Since $a_1 = 5$ and $5 > 3$, the condition $a_{n-1} > 3$ holds for $n \geq 2$. If $a_{n-1} > 3$, then $a_n = 2a_{n-1} - 3 > 2(3) - 3 = 3$. Also, $a_n - a_{n-1} = (2a_{n-1} - 3) - a_{n-1} = a_{n-1} - 3$. Since $a_1 = 5$, $a_n > 3$ for all $n \geq 1$. Thus, $a_n - a_{n-1} = a_{n-1} - 3 > 0$ for $n \geq 2$. This means the sequence is strictly increasing from the first term onwards (5, 7, 11, 19, 35, ...). Therefore, numbers like 8, 16, 22, which are not among the first five terms, cannot appear later in the sequence.

The numbers from the list that are in the sequence are 5, 11, and 35.

The correct answers are Choices A, C, and F.

18. (1 point) In a single line of people waiting to purchase tickets for a movie, there are currently 10 people behind Shandra. If 3 of the people who are currently in line ahead of Shandra purchase tickets and leave the line, and no one else leaves the line, there will be 8 people ahead of Shandra in line. How many people are in the line currently?

☐ people

Solution: Let A be the number of people currently ahead of Shandra. Let B be the number of people currently behind Shandra.

From the problem statement, we know $B = 10$.

We are told that if 3 people from the group ahead of Shandra leave, the number of people ahead of Shandra becomes 8. The number of people currently ahead (A), minus the 3 who leave, equals 8. This can be written as an equation:

$$A - 3 = 8$$

Solving for A by adding 3 to both sides:

$$A = 8 + 3 = 11$$

So, there are currently 11 people ahead of Shandra.

The total number of people in the line is the sum of those ahead of Shandra, Shandra herself, and those behind Shandra.

$$\text{Total People} = (\text{People Ahead}) + (\text{Shandra}) + (\text{People Behind})$$

$$\text{Total People} = A + 1 + B$$

Substitute the values $A = 11$ and $B = 10$:

$$\text{Total People} = 11 + 1 + 10 = 22$$

Therefore, there are currently 22 people in the line.

The correct answer is 22.

19. (1 point) When the decimal point of a certain positive decimal number is moved six places to the right, the resulting number is 9 times the reciprocal of the original number. What is the original number?

Solution: Let the original positive decimal number be n.

Moving the decimal point of n six places to the right is equivalent to multiplying n by 10^6. The resulting number is $n \times 10^6$.

The reciprocal of the original number n is $\frac{1}{n}$.

The problem states that the resulting number ($n \times 10^6$) is equal to 9 times the reciprocal of the original number ($9 \times \frac{1}{n}$). We can write this as an equation:

$$n \times 10^6 = 9\left(\frac{1}{n}\right)$$

To solve for n, first multiply both sides by n:

$$n \times (n \times 10^6) = n \times 9\left(\frac{1}{n}\right)$$

$$n^2 \times 10^6 = 9$$

Now, divide both sides by 10^6:

$$n^2 = \frac{9}{10^6}$$

Finally, take the square root of both sides. Since n must be positive, we only consider the positive root:

$$n = \sqrt{\frac{9}{10^6}}$$

$$n = \frac{\sqrt{9}}{\sqrt{10^6}}$$

Recall that $\sqrt{10^6} = (10^6)^{1/2} = 10^{6 \times 1/2} = 10^3$.

$$n = \frac{3}{10^3}$$

$$n = \frac{3}{1000}$$

As a decimal, this is:

$$n = 0.003$$

The original number is 0.003.

The correct answer is 0.003.

The exam booklet ends here.

Question	Points	Score
1	1	
2	1	
3	1	
4	1	
5	1	
6	1	
7	1	
8	1	
9	1	
10	1	
11	1	
12	1	
13	1	
14	1	
15	1	
16	1	
17	1	
18	1	
19	1	
Total:	19	

1. Ans: (B)
2. Ans: (D)
3. Ans: (B)
4. Ans: (A)
5. Ans: (A)
6. Ans: (C)
7. Ans: (D)
8. Ans: (C)
9. Ans: (D)
10. Ans: (A)
11. Ans: (E)
12. Ans: (B)
13. Ans: (A)
14. Ans: (B)
15. Ans: (D)
16. Ans: (C)
17. Ans: (A), (C), (F)